This document contains a collection of formulas and constants useful for SPC chart construction. It assumes you are already familiar with SPC.

Terminology

Generally, a bar drawn over a symbol means the average of all the values in a sample or group of values. Thus $\bar{x}$ is the average of the x values in a sample.

n – sample size, the number of values in a sample
$\bar{x}$ - average of measurements in a sample
$\bar{\bar{x}}$ - average of averages (of all samples)
R – the range of a sample (difference between largest and smallest values)
$\bar{R}$ - the average of ranges
S – sample standard deviation
$\bar{S}$ - the average of the standard deviations of the samples
$\hat{\sigma}$ - estimate of the process standard deviation (pronounced “sigma hat”)
σ_E - standard error; standard deviation of sample means (pronounced “sigma e”)
CL – center line
UCL – upper control limit
LCL – lower control limit

Control Charts for Variables

$\bar{x}$ and R Charts

Calculating $\bar{x}$ and R

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad R = \max(x_1, x_2, \ldots x_n) - \min(x_1, x_2, \ldots x_n)$$

Estimating population standard deviation

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

Estimating standard deviation of sample means

$$\sigma_E = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{\bar{R}}{d_2 \cdot \sqrt{n}}$$

Calculating Center Line and Control Limits

$$CL_{\bar{x}} = \bar{\bar{x}}$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2 \cdot \bar{R} \qquad LCL_{\bar{x}} = \bar{\bar{x}} - A_2 \cdot \bar{R}$$

$$CL_R = \bar{R}$$

$$UCL_R = \bar{R} \cdot D_4 \qquad UCL_R = \bar{R} \cdot D_3$$

Table of constants

n	A_2	D_3	D_4	d_2
2	1.880	0.0	3.267	1.128
3	1.023	0.0	2.574	1.693
4	0.729	0.0	2.282	2.059
5	0.577	0.0	2.115	2.326
6	0.483	0.0	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078

$\bar{x}$ and S Charts

Calculating $\bar{x}$ and S

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad S = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

Estimating population standard deviation

$$\hat{\sigma} = \frac{\bar{S}}{c_4}$$

Estimating standard deviation of sample means

$$\sigma_E = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{\bar{S}}{c_4 \cdot \sqrt{n}}$$

Calculating Center Line and Control Limits

$$CL_{\bar{x}} = \bar{\bar{x}}$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_3 \cdot \bar{S} \qquad LCL_{\bar{x}} = \bar{\bar{x}} - A_3 \cdot \bar{S}$$

$$CL_S = \bar{S}$$

$$UCL_S = \bar{S} \cdot B_4 \qquad UCL_S = \bar{S} \cdot B_3$$

Table of constants

n	c_4	A_3	B_3	B_4
8	0.9650	1.099	0.185	1.815
9	0.9693	1.032	0.239	1.761
10	0.9727	0.975	0.284	1.716
11	0.9754	0.927	0.321	1.679
12	0.9776	0.886	0.354	1.646
13	0.9794	0.850	0.382	1.618
14	0.9810	0.817	0.406	1.594
15	0.9823	0.789	0.428	1.572
16	0.9835	0.763	0.448	1.552
17	0.9845	0.739	0.466	1.534
18	0.9854	0.718	0.482	1.518
19	0.9862	0.698	0.497	1.503
20	0.9869	0.680	0.510	1.490
21	0.9876	0.663	0.523	1.477
22	0.9882	0.647	0.534	1.466
23	0.9887	0.633	0.545	1.455
24	0.9892	0.619	0.555	1.445
25	0.9896	0.606	0.565	1.435

For values of n greater than 25, the following may be used:

$$c_4 = \frac{4\cdot (n-1)}{4\cdot n-3} \qquad A_3 = \frac{3}{c_4\cdot \sqrt{n}}$$

$$B_3 = 1 - \frac{3}{c_4\cdot \sqrt{2\cdot (n-1)}} \qquad B_4 = 1 + \frac{3}{c_4\cdot \sqrt{2\cdot (n-1)}}$$

X (individuals) and Moving Range (MR) Charts

In this special case n is interpreted as 2 when considering the range, but as 1 when considering the number of values in each sample. Thus d_2, D_3, and D_4 are chosen based on n=2. When calculating the sampling distribution, however, n=1 is used.

Calculating the Moving Range

$MR = |x_i - x_{i-1}|$ (the positive difference between one value and the previous)

Estimating population standard deviation

$\hat{\sigma} = \frac{\overline{R}}{d_2}$ (where $d_2 = 1.128$, since the sample size is 2)

Estimating standard deviation of sample means

$$\sigma_E = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{\overline{MR}}{d_2\cdot \sqrt{n}} = \frac{\overline{MR}}{1.128\cdot \sqrt{1}} = \frac{\overline{MR}}{1.128}$$

Calculating Center Line and Control Limits

$$CL_x = \bar{x}$$

$$UCL_{\bar{x}} = \bar{x} + 3\cdot \frac{\overline{MR}}{d_2} = \bar{x} + 2.660\cdot \overline{MR} \qquad LCL_{\bar{x}} = \bar{x} - 3\cdot \frac{\overline{MR}}{d_2} = \bar{x} - 2.660\cdot \overline{MR}$$

$CL_{MR} = \overline{MR}$ where $\overline{MR} = \frac{\sum_{i=1}^{k} MR}{k-1}$ and k is the number of samples taken.

$$UCL_{MR} = \overline{MR}\cdot D_4 = \overline{MR}\cdot 3.267 \qquad LCL_{MR} = \overline{MR}\cdot D_3 = 0$$

Control Charts for Attributes

Chart for proportion defective - p Charts

Requires constant sample size.

Proportion defective $p = \frac{D}{n}$

$CL = \overline{p}$

$UCL = \overline{p} + 3 \cdot \sqrt{\frac{\overline{p} \cdot (1 - \overline{p})}{n}}$

$LCL = \overline{p} - 3 \cdot \sqrt{\frac{\overline{p} \cdot (1 - \overline{p})}{n}}$

Chart for number defective – np Chart

Allows for variable sample size with varying control limits.

Number defective np

$CL = n \cdot \overline{p}$

$UCL = n \cdot \overline{p} + 3 \cdot \sqrt{n \cdot \overline{p} \cdot (1 - \overline{p})}$

$LCL = n \cdot \overline{p} - 3 \cdot \sqrt{n \cdot \overline{p} \cdot (1 - \overline{p})}$

Chart for number of defects/nonconformances per sampling unit – c Chart

(The subtlety here is that in c and u charts, a single unit may have more than one defect, while with p and np charts, an item is either defective or not.)

$CL = \overline{c}$

$UCL = \overline{c} + 3 \cdot \sqrt{\overline{c}}$

$LCL = \bar{c} - 3 \cdot \sqrt{\bar{c}}$

<u>Chart for number of defects/nonconformances per sampling unit - u Charts</u>

Allows for variable size of sampling unit with variable control limits.

$u = \frac{x}{n}$

$CL = \bar{u}$

$UCL = \bar{u} + 3 \cdot \sqrt{\frac{\bar{u}}{n}}$

$LCL = \bar{u} - 3 \cdot \sqrt{\frac{\bar{u}}{n}}$

Sensitizing Rules for Control Charts

Normally, a single point outside the control limits is considered to signal an out of control process. Under some circumstances, however, such as while working to establish statistical control, it is desirable to employ "sensitizing rules" which make it more likely that a small change in mean or variability will be detected.

The so-called "Western Electric Company" or "WECO" rules include:

- One point outside the control limits
- Two of three consecutive points outside the two sigma zone (on the same side of center)
- Four of five consecutive points outside the one sigma zone (on the same side of center)
- Eight consecutive points on the same side of the center line

Additional commonly used sensitizing rules:

- Six consecutive points steadily increasing or decreasing
- Fifteen consecutive points inside the one sigma zone
- Fourteen consecutive points alternating up and down
- Eight consecutive points outside the one sigma zone
- A clearly non-random pattern

Process Capability Analysis

Capability Indices

$$C_p = \frac{USL - LSL}{6 \cdot \sigma}$$

C_{pk} is the lesser of:

$$C_{pu} = \frac{USL - \mu}{3 \cdot \sigma}$$

$$C_{pl} = \frac{\mu - LSL}{3 \cdot \sigma}$$

where if μ and σ are not known, they can be replaced with their estimates $\hat{\mu}$ and $\hat{\sigma}$, e.g. $\bar{\bar{x}}$ and $\frac{\bar{R}}{d_2}$ or $\frac{\bar{S}}{c_4}$

(While the foregoing is believed to be correct and accurate, it is provided with no warranty whatever.)

